CS 13: Mathematics for Computer Scientists — Spring 2023

Written Homework 00 (due Friday, April 14)

Directions: *Write up carefully argued solutions to the following problems. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use results from lecture and previous homeworks without proof.*

0. Your Average Induction (25 points)

Consider the following code:

```
def average(grades, num_grades):
   if num_grades == 1:
      return grades[0]
   else:
      return (
         (average(grades, num_grades - 1) * (num_grades - 1)) +
         grades[num_grades - 1]
      ) / num_grades
```

Let A be an arbitrary, non-empty `list[int]`, and let $n = \texttt{len}(A)$.

Prove that $\texttt{average}(A, n) = \sum_{i=0}^{n-1} \frac{A_i}{n}$, where $A_i = A[i]$ for $0 \leq i < n$ for all $n \geq 1$.

1. No, You're Being Irrational (20 points)

Prove that $\sqrt{2} + \sqrt{5}$ is irrational. You may use the fact that $\sqrt{2}$ is irrational, but you do not have to.

2. Prime Examples (25 points)

Prove that for any prime $p > 3$, either $p \equiv_6 1$ or $p \equiv_6 5$.

3. Balanced Ternary (30 points)

In the balanced ternary number system, numbers are made up of "trits": $\{0, 1, T\}$. To evaluate a trit, we use a "valuation function", called V, defined as follows:

$$V(X) = \begin{cases} 0 & \text{if } X = 0 \\ 1 & \text{if } X = 1 \\ -1 & \text{if } X = T \end{cases}$$

To evaluate an entire balanced ternary number, a summation is used as usual:

$$\text{evaluate}_n(t_{n-1}t_{n-2}\ldots t_0) = \sum_{i=0}^{n-1} V(t_i) \cdot 3^i$$

Prove that $\text{evaluate}_n(X)$ is injective for all n. That is, prove that no two (different) inputs lead to the same output.