CS 13: Mathematical Foundations of Computing

Lecture 7 Exercises

Name:

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0.

Functions on Trees

$$\begin{aligned} \texttt{flip}(\texttt{Nil}) &= \texttt{Nil} \\ \texttt{flip}(\texttt{Tree}(x,\ L,\ R)) &= \texttt{Tree}(x,\ \texttt{flip}(R),\ \texttt{flip}(L)) \end{aligned}$$

Claim

For all $T \in$ **Tree**,

$$\texttt{flip}(\texttt{flip}(T)) = T$$

1.

Functions on Trees

$$\begin{aligned} \texttt{full}(0) &= \texttt{Nil} \\ \texttt{full}(n) &= \texttt{Tree}(n,\ \texttt{full}(n-1),\ \texttt{full}(n-1)) \end{aligned}$$

$$\begin{aligned} \texttt{num\_leaves}(\texttt{Nil}) &= 1 \\ \texttt{num\_leaves}(\texttt{Tree}(x,\ L,\ R)) &= \texttt{num\_leaves}(L) + \texttt{num\_leaves}(R) \end{aligned}$$

Claim

For all $n \in \mathbb{N}$,

$$\texttt{num\_leaves}(\texttt{full}(n)) = 2^n$$

2.

Functions on Trees

$$\begin{aligned} \texttt{size}(\texttt{Nil}) &= 0 \\ \texttt{size}(\texttt{Tree}(x,\ L,\ R)) &= 1 + \texttt{size}(L) + \texttt{size}(R) \end{aligned}$$

$$\begin{aligned} \texttt{height}(\texttt{Nil}) &= 0 \\ \texttt{height}(\texttt{Tree}(x,\ L,\ R)) &= 1 + \texttt{max}(\texttt{height}(L), \texttt{height}(R)) \end{aligned}$$

Claim

For all $T \in$ **Tree**,

$$\texttt{size}(T) \leq 2^{\texttt{height}(T)} - 1$$