CS 13: Mathematical Foundations of Computing

Lecture 6 Exercises

Name:

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Functions on Lists

$$\begin{aligned} \texttt{len}([]) &= 0 \\ \texttt{len}(x :: L) &= 1 + \texttt{len}(L) \\ \\ \texttt{sum}([]) &= 0 \\ \texttt{sum}(x :: L) &= x + \texttt{sum}(L) \end{aligned}$$

Claim

For all $L \in$ **List**, where the list does not contain 0,

$$\texttt{sum}(L) \geq \texttt{len}(L)$$

1.

Functions on Lists

$$\begin{aligned} \texttt{sum}(\texttt{[]}) &= 0 \\ \texttt{sum}(x :: L) &= x + \texttt{sum}(L) \end{aligned}$$

$$\begin{aligned} \texttt{sum2}(\texttt{acc},\ \texttt{[]}) &= \texttt{acc} \\ \texttt{sum2}(\texttt{acc},\ x :: L) &= \texttt{sum2}(\texttt{acc} + x,\ L) \end{aligned}$$

Claim

For all $L \in$ **List** and $\texttt{acc} \in \mathbb{N}$,

$$\texttt{sum(L)} + \texttt{acc} = \texttt{sum2}(\texttt{acc},\ L)$$

2.

Functions on Lists

$$\begin{aligned} \texttt{append}(\texttt{[]},\ a) &= a :: \texttt{[]} \\ \texttt{append}(x :: L,\ a) &= x :: \texttt{append}(L, a) \end{aligned}$$

$$\begin{aligned} \texttt{removeLast}(\texttt{[]}) &= \texttt{[]} \\ \texttt{removeLast}(x :: L) &= \textbf{if } L \texttt{ == [] } \textbf{then } \texttt{[] } \textbf{else } x :: \texttt{removeLast}(L) \end{aligned}$$

Claim

For all $L \in$ **List** and $a \in \mathbb{N}$,

$$\texttt{removeLast}(\texttt{append}(L, a)) = L$$