

**Lecturcises 03 (due Saturday, October 21 @ 11:30pm)**

**Directions:** *These problems were presented within the last week as “exercises” in lecture. During lecture, you were able to collaborate with students, TAs, and Prof. Blank. Your task now is to write up solutions to these problems **without discussing them with anyone**. You should submit the lecturcise below on Gradescope. Note that your submissions will be graded on correctness, not effort.*

**Height and size**

Let

$$\text{size}(\text{Nil}) = 0$$

$$\text{size}(\text{Tree}(x, L, R)) = 1 + \text{size}(L) + \text{size}(R)$$

$$\text{height}(\text{Nil}) = 0$$

$$\text{height}(\text{Tree}(x, L, R)) = 1 + \max(\text{height}(L), \text{height}(R))$$

Show that for all trees  $T$ ,  $\text{size}(T) \leq 2^{\text{height}(T)} - 1$ .