CS 13: Mathematical Foundations of Computing — Fall 2023

Lecturcises 01 (due Saturday, October 7 @ 11:30pm)

Directions: *These problems were presented within the last week as "exercises" in lecture. During lecture, you were able to collaborate with students, TAs, and Prof. Blank. Your task now is to write up solutions to these problems* ***without discussing them with anyone****. You should submit* ***exactly TWO*** *of the lecturcises below on Gradescope. Note that your submissions will be graded on* correctness*, not effort. Some of these results appear in the Definitions and Theorems handout for use in future proofs - proofs which cite this exact result without any non-trivial manipulation will earn a 0.*

Mon, Oct 2

The following are the lecturcises presented in lecture on Monday.

Unary surjectivity

Let $\Sigma = \{\triangle\}$ and define the unary valuation function $V : \Sigma^+ \to \mathbb{N} \setminus \{0\}$, as follows:

$$V(\triangle) = 1$$
$$V(\triangle X) = 1 + V(X)$$

Prove that V is surjective.

Binary evaluation

Let $\Sigma = \{0, 1\}$ and define the binary valuation function $V : \Sigma^+ \to \mathbb{N}$, as follows:

$$V(b) = b$$
$$V(Xb) = 2V(X) + b$$

Prove the closed form for V,

$$V(b_{n-1}b_{n-2}\dots b_0) = \sum_{i=0}^{n-1} 2^i b_i$$

Wed, Oct 4

The following are the lecturcises presented in lecture on Wednesday.

Exercises will be added after lecture.

Fri, Oct 6

The following are the lecturcises presented in lecture on Friday.

Congruence and mod equivalence

Let a and b be integers, and let m be a positive integer. Then, $a \equiv_m b$ iff $a \bmod m = b \bmod m$.

Adding congruences

Let $a, b, c, d \in \mathbb{Z}$, and let m be a positive integer. Then, if $a \equiv_m b$ and $c \equiv_m d$, then $a + c \equiv_m b + d$.

Multiplying congruences

Let Let $a, b, c, d \in \mathbb{Z}$, and let m be a positive integer. Then, if $a \equiv_m b$ and $c \equiv_m d$, then $ac \equiv_m bd$.