

CS 13: Mathematical Foundations of Computer Science

Set Definitions

Common Sets

- $\mathbb{N} = \{0, 1, 2, \dots\}$ is the set of *Natural Numbers*.
- $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ is the set of *Integers*.
- $\mathbb{Q} = \left\{ \frac{p}{q} : p, q \in \mathbb{Z} \wedge q \neq 0 \right\}$ is the set of *Rational Numbers*.
- \mathbb{R} is the set of *Real Numbers*.

Containment, Equality, and Subsets

Let A, B be sets. Then:

- $x \in A$ (" x is an *element* of A ") means that x is an element of A .
- $x \notin A$ (" x is *not* an *element* of A ") means that x is *not* an element of A .
- $A \subseteq B$ (" A is a *subset* of B ") means that all the elements of A are also in B .
- $A \supseteq B$ (" A is a *superset* of B ") means that all the elements of B are also in A .
- $(A = B) \equiv (A \subseteq B) \wedge (B \subseteq A) \equiv \forall x (x \in A \leftrightarrow x \in B)$

Set Operations

Let A, B be sets. Then:

- $A \cup B$ is the *union* of A and B . $A \cup B = \{x : x \in A \vee x \in B\}$.
- $A \cap B$ is the *intersection* of A and B . $A \cap B = \{x : x \in A \wedge x \in B\}$.
- $A \setminus B$ is the *difference* of A and B . $A \setminus B = \{x : x \in A \wedge x \notin B\}$.
- $A \oplus B$ is the *symmetric difference* of A and B . $A \oplus B = \{x : x \in A \oplus x \in B\}$.
- \overline{A} is the *complement* of A . If we restrict ourselves to a "universal set", \mathcal{U} , (a set of all possible things we're discussing), then $\overline{A} = \{x \in \mathcal{U} : x \notin A\}$.

Set Constructions

Let A, B, C, D be sets. Then:

- $S = \{x : P(x)\}$ is *set comprehension notation* which means S is a set that contains all objects x with property P .
- $A \times B$ is the *cartesian product* of A and B . $A \times B = \{(a, b) : a \in A, b \in B\}$.
- $[n]$ ("*brackets n* ") is the set of natural numbers from 1 to n . $[n] = \{x \in \mathbb{N} : 1 \leq x \leq n\}$.
- $\mathcal{P}(A)$ is the *power set* of A . $\mathcal{P}(A) = \{S : S \subseteq A\}$.